THE ROLE OF MATHEMATICAL CONCEPTS IN REAL-WORLD PROBLEMS

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Annotation. This thesis shows how mathematics applies to real-life problems. It includes designing a sustainable box using calculus, solving the Stable Matching Problem with algorithms, and finding a dam's area using integrals, highlighting math's role in solving practical challenges.

Introduction. This paper presents three real-world applications of mathematics that demonstrate how abstract concepts can solve practical problems. The first case applies calculus to design a sustainable package using minimal material. The second explores computational thinking through the Stable Matching Problem, showing how algorithms can ensure fairness in assignments. The third uses integral calculus to determine the area of a gravity dam's cross-section, combining geometry and integration. Together, these examples highlight the versatility of mathematical tools in optimizing design, supporting decision-making, and solving engineering challenges efficiently.

Rational function for sustainable package: a case study (Calculus)

Imagine a packaging company needs to design a shipping box with a square base and a volume of 600 cubic inches. What dimensions should it have? Should the base be 5×5 inches, 10×10 inches, or something else? Which one uses the least material? (1 inch = 2.54 cm)

Solution:

We know the volume of the box must be 600 cubic inches, so:

$$600 = l^2 \cdot h$$

Now, substitute this into the surface area formula:

$$S = 4l \cdot \frac{600}{l^2} + 2l^2 = 2l^2 + \frac{2400}{l}$$

Let's now **find the minimum surface area** using calculus by taking the derivative of the surface area function:

$$S(l) = 2l^2 + \frac{2400}{l}$$

$$\frac{dS}{dl} = \frac{d}{dl}\left(2l^2 + \frac{2400}{l}\right) = 4l - \frac{2400}{l^2}.$$

To find critical points (possible minimums), we will solve:

$$4l - \frac{2400}{l^2} = 0$$

$$l = \sqrt[3]{600} = 8.43$$

Second derivative test to check if it is a minimum

$$\frac{d^2S}{dl^2} = \frac{d}{dl}\left(4l - \frac{2400}{l^2}\right) = 4 + \frac{4800}{l^3}$$

Since this is always positive for $l > 0$, the function has a minimum at $l = 8.43$. From this we can always find $h = 8.43$.

This means the **box is a cube** with all sides approximately 8.43 **inches**, and it uses the least material for a fixed volume of 600 cubic inches.

The Stable Matching Problem (Computational Thinking)

Every year, thousands of students graduate from medical schools and begin their careers by applying for residency training positions in hospitals. Each medical student has a personal ranking of the hospitals where they would prefer to work, and each hospital has its own ranking of students based on academic merit, interviews, and institutional fit.

Students: John, Alice, Maria, Ethan, Noor.

Hospitals: Hospital A, Hospital B, Hospital C, Hospital D, Hospital E.

Student	1st	2nd	3rd	4th	5th
John	B	A	D	E	C
Alice	A	C	B	D	E
Maria	B	D	E	A	C
Ethan	C	B	A	D	E
Noor	D	E	C	B	A

Hospital	1st	2nd	3rd	4th	5th
A	Ethan	Alice	John	Maria	Noor
B	John	Maria	Alice	Noor	Ethan
C	Ethan	Noor	Alice	John	Maria
D	Noor	Maria	Ethan	Alice	John
E	Alice	Ethan	Noor	Maria	John

Solution. Let's solve using **Gale–Shapley.**

Step 1: Everyone proposes to 1st choice

John → B

Alice → A

Maria → B

Ethan → C

Noor → D

Hospitals evaluate: B: John, Maria → prefers John (1st), rejects Maria

A: Alice → accepts

C: Ethan → accepts

D: Noor → accepts

Step 2: Maria proposes to 2nd choice

D already holds Noor. D ranks: Noor (1st), Maria (2nd) → keeps Noor, rejects Maria again.

Step 3: Maria proposes to 3nd choice: E is free, accepts Maria.

Final matching:

Student	Hospital
John	B
Alice	A
Maria	E
Ethan	D
Noor	C

Area of the Gravity Dam (Calculus)

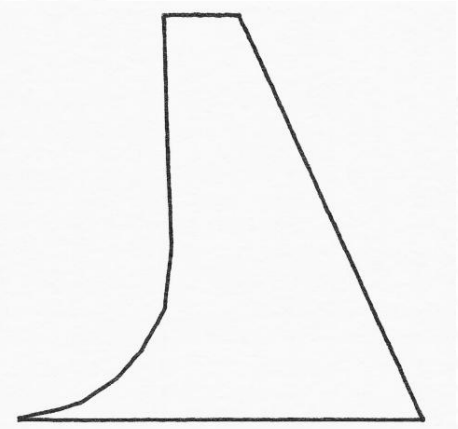

Image 1.

You can see that the **gravity dam**'s cross-section is composed of **three fundamental geometric shapes**: a rectangle, a right triangle, an area under the parabola.

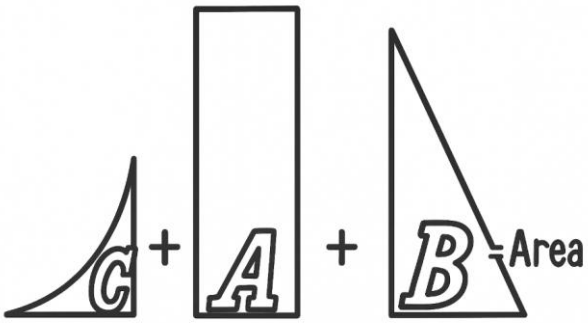

Image 2.

$$Area_A = h \cdot b = 389 \cdot 16 = 6224\, ft^2$$

$$Area_B = \frac{1}{2} h \cdot a = \frac{1}{2} 389 \cdot 59 = 11475\, ft^2$$

We can break it into these three pieces. We can use simple geometry to calculate the area of the rectangular piece and the triangular piece, which leaves us with calculating the area of the parabolic piece. This is where the calculus will be used.

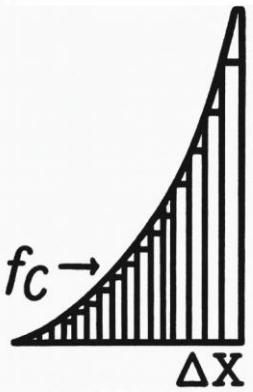

Image 3.

We are using integral calculus to calculate this. If we keep making the rectangles smaller and smaller, we will eventually get the exact area under the curve. The curve starts at the origin $(0,0)$ and rises to a height of $389\, ft$ at the end of the $54\, ft$ wide base.

To model the curve, we use a simple quadratic function:

$$f_C(x) = ax^2$$

Using the known point $(54,389)$, we find the value of a:

$$a = \frac{389}{2916}.$$

So the function becomes:

$$f_C(x) = \frac{389}{2916} x^2.$$

Now we will use Riemann integral:

$$Area_C = \lim_{|\Delta x_i| \to 0} \sum_i f_C(x_i)\Delta x_i = \int_a^b f_C(x)dx$$

$$Area_C = \int_0^{54} f_C(x)dx = \int_0^{54} \frac{389}{2916}x^2 dx = \frac{389}{2916} \cdot \frac{54^3}{3} \approx 5803\, ft^2$$

$$Total Area = 23502\, ft^2.$$

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